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**THEORETICAL PREDICTION OF THE EQUILIBRIUM
REAL GAS INVISCID FLOW FIELDS ABOUT BLUNT BODIES**

Final Phase II Formal Written Report

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**RESEARCH AND ADVANCED DEVELOPMENT DIVISION
AVCO CORPORATION**

Wilmington, Massachusetts

under subcontract RL - 30107

December 1963

submitted to

**NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
MANNED SPACECRAFT CENTER
GENERAL RESEARCH PROCUREMENT OFFICE**

Houston 1, Texas

under Contract No. NAS 9 - 858

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I. INTRODUCTION

Under Subcontract RL-30107 of Contract NAS 9-858, Avco-RAD has undertaken a study of various methods of solution of the flow fields over blunt bodies. Interim results of this study were presented in Reference 1. A program for the determination of the flow of equilibrium air over a blunt body at zero angle of attack has been submitted to NASA MSC and is described in Reference 9. A comparative analysis of the methods of integration in current use led to the selection of the method of integral relations as most suitable for the determination of three-dimensional flow fields. A computational procedure employing the method of integral relations has been developed which is relevant to the moderate angle-of-attack range of interest.

The conclusions of the comparison of integration methods and the formulation of the method of integral relations are presented in this final report. Attention has been given to the configuration of the dividing streamline, which yields sufficient information for the integration procedure to be properly posed. Emphasis has been placed on obtaining a relatively simple method of computation in order to minimize programming difficulties, while at the same time retaining a level of approximation adequate for engineering purposes.

II. REVIEW OF TWO-DIMENSIONAL AND AXISYMMETRIC SOLUTIONS

Several approaches to the blunt body flow problem at zero angle of attack are established theoretically and have been reviewed to determine their suitability for the angle-of-attack problem. Most of these approaches are summarized in Chapter 6 of Hayes and Probst²ein. They consist of various approximate techniques for integrating the inviscid equations of flow in the subsonic or subsonic-supersonic region. A complete integration covers the flow field up to the limiting characteristic - that is, the characteristic (or characteristics) farthest downstream which intersects the sonic line - since disturbances in the supersonic flow may otherwise affect the subsonic region through the sonic line. In order to provide input for method of characteristics calculations, it is necessary to extend the integration somewhat downstream of the limiting characteristic in order to avoid computational difficulties near Mach 1.

Expansion techniques valid near the axis of symmetry and the like are useful, but many terms in the expansion series are required to give sufficient detail for flow field prediction. The inverse problem has been treated successfully by several authors. In this approach, a shock shape is assumed and the integration is then carried out as if the problem were of the initial value type. Using initial conditions computed at the specified shock, the equations are numerically integrated in some manner until the computed value of the stream function vanishes, say; this locus of points is the resulting body shape. Since such a procedure is not mathematically stable when applied

to elliptic (in this case, subsonic) systems of equations, either a smoothing technique must be employed to remove oscillations and irregularities from the solution, or some other method must be used to avoid the instability.

Zlotnick and Newman³, among several papers^{4,5,6} employing the same basic concept, solved the inverse problem by a straightforward finite-difference method. They smooth out unwanted oscillations by systematic fitting of polynomials. Vaglio-Laurin and Ferri⁴ also treat the "transonic" region between the sonic line and the limiting characteristic in a consistent manner by integrating along characteristics.

Garabedian and Lieberstein^{7,8}, an adaptation of whose method for equilibrium air appears in Reference 9 and has been submitted to NASA MSC, introduce a complex transformation which, in a sense, alters the nature of the equations of motion from elliptic to hyperbolic, thereby circumventing the instability problem. The integration from the shock to the unknown body is carried out in the complex domain, and the results are then projected onto the real plane. The stability of the procedure is rooted in the possibility of analytically continuing the functions into the complex domain.

The direct problem, in which the body shape is given and the flow field, including the shock, is solved for, has been treated by Emmons et al.¹⁰ using a streamtube technique. Initial approximations to the shock shape and body pressure distribution are assumed, and the resulting flow field is determined by the streamtube relations. An iterative scheme is employed to correct the shock shape and pressure distribution until the difference

between two successive solutions is slight.

All the methods described above can in principle be generalized to the case of a blunt body at angle of attack. A perturbation technique can be formulated for small angles of attack, as has been done by Vaglio-Laurin and Ferri⁴ for the finite-difference marching procedure, although they present no results. Swigart¹¹ perturbs the equations of motion in the small angle of attack and at the same time develops a solution in the form of a series expansion valid near the axis of symmetry of the shock wave (i.e., near the stagnation line). The angle of attack perturbation permits the dependence on the meridional coordinate to be eliminated, and the series expansion results in a set of ordinary differential equations that are integrated inward from the given paraboloidal shock. Swigart's results for zero angle of attack agree well with experimental and other theoretical results for spherical bodies and ellipsoidal bodies of moderate eccentricity, even up to the sonic point; however, his expansion procedure would not be expected to be accurate for blunt bodies with relatively small corner radii, unless many terms in the series were retained.

Swigart observes that his computed body (dividing) streamline at angle of attack differs from the streamline that passes through the point where the shock is normal to the free stream vector--in other words, the body entropy is not the maximum entropy in the flow field. The important question of the body entropy is still a matter of controversy and is discussed in Sections IV and V.

The direct solution of Belotserkovskii^{12,13} is based on the method of integral relations formulated by Dorodnitsyn¹⁴. The flow field between the body and the shock is split up into a number of equally spaced strips, and the equations of motion are integrated from the body to each strip boundary. Polynomial expressions are assumed for the dependent variables in the direction normal to the body; upon substitution in the integrated equations, there is obtained a set of ordinary differential equations for the coefficients of the polynomials in terms of the coordinate along the body. These equations are solved with assumed values of the stagnation point standoff distance and the initial velocities at the strip boundaries, and the solution is iterated so as to satisfy properly singularities which occur at the sonic line. Traugott¹⁵ and Holt¹⁶ have presented versions of this method.

The papers of Bazzhin¹⁷ and Minailos¹⁸ represent important recent Soviet work in the field of blunt body flow at large angles of attack. Both apply the method of integral relations with one strip in the direction normal to the body, Bazzhin to the two-dimensional flat plate problem and Minailos to the problem of a yawing axisymmetric body. Vaglio-Laurin¹⁹, in addition to his treatment of the application of the PLK method to the calculation of blunt-body flows, presents an analysis similar to Bazzhin's for asymmetric two-dimensional shapes. Both Soviet authors make allowances for the possibility that the dividing streamline does not pass through the normal point of shock,

but do not furnish the additional condition necessary for its determination. Bazzhin summarizes the results of his calculations, which were performed with the initial assumption that the dividing streamline did pass through the normal point on the shock. He notes that for angles of attack sufficiently large ($> 30^\circ$) that his resulting solution indicated that the initial assumption on the dividing streamline was incorrect, it was not possible to iterate the solution in order to obtain successive values of the entropy on the body.

A comparison of the various methods developed for zero angle of attack flows, with a view to determining the most suitable approach for the calculation of flows past axisymmetric bodies at large angles of attack, reveals that the inverse methods are conceptually simplest. With the problem of mathematical stability almost completely resolved, there is little difficulty in sketching the broad outlines of an inverse technique, in which the equations of motion are integrated in a step-by-step marching procedure starting from the given shock and working toward the unknown body. A look into the details of the numerical method, however, is rather discouraging. A preliminary investigation of the Garabedian and Lieberstein method, for example, showed that the problems involved in formulating a method for integrating the equations in the complex domain were much more formidable than expected, and the approach was abandoned. Essentially a three-dimensional finite-difference method would have been required. Such a method would

have tied up considerable computing machine storage capacity and would have been difficult to program, inasmuch as no previous programming experience on problems of that type was available. It was also recognized that the selection of the proper shock shape associated with a desired body would require a certain amount of experience, particularly for large angles of attack. The same objections applied to other inverse techniques. Although it would undoubtedly be possible to program one of these techniques, it would suffer from the same difficulties associated with the three-dimensional method of characteristics - excessive computing time, the laborious specification of input, and the constant attention of someone closely familiar with the program.

Attention therefore has been centered on the direct methods. The streamtube method appears undesirable both from the aspect of its formulation for three dimensions and from certain computational problems caused by the lack of accuracy in computing streamline curvatures numerically. The method of integral relations has the advantage that the variation of properties in the direction normal to the surface is taken into account through expansion in series and integration, so that there remains only the variation in two surface directions. Furthermore, the meridional variation can be approximated by Fourier series. Finally, the method allows a systematic treatment of the sonic surface, and the integration can be extended downstream to provide input for calculations of the supersonic flow. For these reasons, the method of integral relations has been adopted for the three-dimensional flow problem.

III. CONFIGURATION OF THE DIVIDING STREAMLINE

Consider a cylindrical coordinate system (n, r, θ) with velocity components (v, u, w) , located at the stagnation point with n - axis perpendicular to the surface (Figure 1).

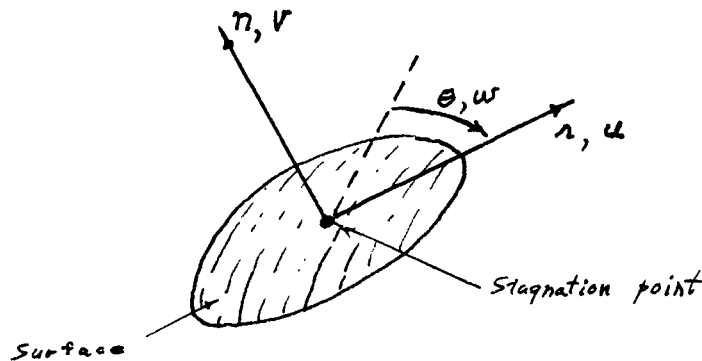


Figure 1. Configuration of Coordinates at the Stagnation Point

Expanding in Taylor series,

$$\begin{aligned} u &= a(\theta)r + d(\theta)n, \\ w &= b(\theta)r + e(\theta)n, \\ v &= f(\theta)r + c(\theta)n. \end{aligned} \quad (1)$$

But $f(\theta) = 0$ because $v = 0$ at $n = 0$, and $c(\theta) = c = \text{constant}$ because otherwise v would not be single-valued along the n -axis.

The vorticity

$$\vec{\Omega} = \left[\frac{(rw)_n - u_\theta}{r}, \frac{v_\theta}{r} - w_n, v_n - u_n \right] \quad (2)$$

in the (n, r, θ) directions, or

$$\vec{\Omega} = \left[2b - a_\theta + (e - d_\theta)\frac{n}{r}, -e, -d \right]. \quad (3)$$

Since the n -component must be finite at $r = 0$, $e - d_\theta = 0$; then $2b - a_\theta = \text{constant}$, since otherwise the vorticity would not be single-valued.

The momentum equation gives $p_n = p_r = p_\theta = 0$ because of the vanishing velocity, and similarly the equation of conservation of stagnation enthalpy gives $h_n = h_r = h_\theta = 0$. Hence all thermodynamic variables are stationary at the stagnation point. The continuity equation then gives

$$\nabla \cdot \vec{q} = 2a + b_\theta + c = 0 \quad (4)$$

and the vorticity equation gives

$$\begin{aligned} \nabla \times (\vec{q} \times \vec{Q}) = & \left[-(2a + b_\theta)(2b - a_\theta) \right. \\ & - \left\{ (2b - a_\theta)(d + e_\theta) + c(e + d_\theta) \right\} \frac{n}{r}, \\ & (2b - a_\theta)d + ce + (be - ad)_\theta + (e^2 - d^2)_\theta \frac{n}{r}, \\ & \left. (b - a_\theta)e + (a + c)d \right] = 0. \end{aligned} \quad (5)$$

Hence

$$-(2a + b_\theta)(2b - a_\theta) = c(2b - a_\theta) = 0,$$

or

$$2b - a_\theta = 0. \quad (6)$$

Also

$$(2b - a_\theta)(d + e_\theta) + c(e + d_\theta) = 0,$$

or

$$e + d_\theta = 0. \quad (7)$$

But we showed above that $e - d_{\theta} = 0$. Hence $e = d_{\theta} = 0$. We also have $(b - a_{\theta})e + (a + c)d = 0$, which gives $d = 0$.

Therefore, all three components of the vorticity at the stagnation point are zero, and the velocity components in the neighborhood of the stagnation point have the form

$$\begin{aligned} u &= a(\theta)r, \\ w &= b(\theta)r, \\ v &= c\eta. \end{aligned} \tag{8}$$

Thus the outflow velocity u and the cross flow w vanish along the η -axis ($r = 0$) perpendicular to the body, and the stagnation streamline is perpendicular to the surface.

IV. FORMULATION OF THREE-DIMENSIONAL PROGRAM FOR EQUILIBRIUM AIR

The coordinate system and basic equations employed in the one-strip analysis for angles of attack have been presented in Reference 1 and will not be repeated here. The same notation will be used in the present report.

There follows an outline of the final equations and integration method appropriate for the one-strip method. In the derivation of the initial conditions, use was made of the fact that the dividing streamline is perpendicular to the surface at the stagnation point (see the previous section). In the one-strip approximation, the dividing streamline becomes a straight line perpendicular to the body. This can be seen from the fact that the velocity component u lacks any linear dependence on x in the vicinity of the stagnation point, so that, to be consistent with the linear approximation, it must vanish identically along the stagnation point normal. In addition, the fundamental differential equation for u_x cannot be satisfied at the stagnation point unless the value of u at the shock point opposite the stagnation point vanishes.

It was therefore concluded that the initial shock angle must be such that $u_{s_0} = 0$, and that the body entropy has the value associated with that shock angle. The problem then essentially reduces to the determination of the proper values of two parameters, the stagnation point location and initial shock detachment distance, which will allow the solution to be continued through the two sonic point singularities.

In the following equations, $g = \sqrt{g}$. In place of the variables u, y (both ≥ 0) and δ ($\leq \pi/2$), it was found to be more convenient to substitute the variables U, Y, Δ defined by

$$Y = \begin{cases} +y & \text{above body axis} \\ -y & \text{below body axis} \end{cases} \quad (9)$$

(in other words, a cartesian rather than cylindrical coordinate),

$$\Delta = \begin{cases} \delta & \text{above body axis} \\ \pi - \delta & \text{below body axis,} \end{cases} \quad (10)$$

and

$$U = \begin{cases} +u & \text{for } Y \geq R_1 \cos \Delta_0 \\ -u & \text{for } Y < R_1 \cos \Delta_0, \end{cases} \quad (11)$$

where $\Delta_0 = \pi - \delta_0$. Also define $\Delta_1 = \pi - \delta_1$. The function W is given by

$$W = \frac{\partial w}{\partial \theta} = \pm \left[\left\{ \frac{U(Y) + U(-Y + 2R_1 \cos \Delta_0)}{2} \right\}^2 + \left\{ \frac{U(Y) - U(-Y + 2R_1 \cos \Delta_0)}{2} \right\}^2 \right]^{1/2}, \quad (12)$$

where the plus sign is selected for $Y < R_1 \cos \Delta_0$ and the minus sign for $Y \geq R_1 \cos \Delta_0$.

The equations of motion reduce to three ordinary differential equations for U, λ (in this case an implicit function of the variables at the shock), and ϵ :

$$\begin{aligned} \left[\left(1 - \frac{U^2}{a^2}\right) g \rho \sin \Delta \frac{dU}{dY} \right]_s &= - \left[\frac{\rho U}{g} \sin \Delta \frac{d}{dY} (\epsilon g) + \rho W \right]_s \\ - \epsilon \left[\sin \Delta \frac{d}{dY} \left\{ \left(\frac{1}{\epsilon} + \frac{1}{R_0}\right) g \rho U \right\} + \left(\frac{1}{\epsilon} + \frac{1}{R_1}\right) \left\{ 2 \left(\frac{1}{\epsilon} + \frac{1}{R_0}\right) g \rho U + \rho W \right\} \right]_s & \\ \equiv F, \end{aligned} \quad (13)$$

$$\sin \Delta \frac{d}{dY} \left[\left(\frac{1}{\epsilon} + \frac{1}{R_\theta} \right) g \rho U v \right]_s = \frac{g}{\epsilon} \left[\frac{\rho U^2}{R_\lambda} + \left(\frac{2}{\epsilon} + \frac{1}{R_\lambda} + \frac{1}{R_\theta} \right) \rho \right]_s \quad (14)$$

$$+ \left[\left(\frac{1}{\epsilon} + \frac{1}{R_\theta} \right) g \left\{ \frac{\rho U^2}{R_\lambda} - 2 \left(\frac{1}{\epsilon} + \frac{1}{R_\lambda} \right) \rho v^2 \right\} - \left(\frac{1}{\epsilon} + \frac{1}{R_\lambda} \right) \rho v W \right. \\ \left. - \left(\frac{2}{\epsilon} + \frac{1}{R_\lambda} + \frac{1}{R_\theta} \right) \frac{g \rho}{\epsilon} \right]_s ,$$

$$\sin \Delta \frac{d\epsilon}{dY} = \left(1 + \frac{\epsilon}{R_\lambda} \right) \tan \lambda \quad (15)$$

Here

$$h_t = H - \frac{1}{2} U_t^2 \quad (H = h_\infty + \frac{1}{2} V_\infty^2), \quad (16)$$

$$h_s = H - \frac{1}{2} (U_s^2 + v_s^2), \quad (17)$$

$$\left. \begin{aligned} \rho_t &= \rho(h_t, S_t) \\ \rho_v &= \rho(h_t, S_t) \end{aligned} \right\} \quad \begin{aligned} &\text{(equation of state of equilibrium air given by curve} \\ &\text{fits with } S_t = \text{constant),} \end{aligned} \quad (18)$$

$$a_t = a(h_t, S_t) \quad (\text{curve-fit speed of sound}). \quad (19)$$

The functions with subscript "s" are evaluated in terms of the variables λ by the equations

$$U_s = V_N \sin \lambda + V_\infty \cos \lambda \cos \nu, \quad (20)$$

$$V_S = -V_N \cos \lambda + V_\infty \sin \lambda \cos v, \quad (21)$$

$$\rho_S = \frac{\rho_\infty V_\infty \sin v}{V_N}, \quad (22)$$

$$p_S = p_\infty + \rho_\infty V_\infty \sin v (V_\infty \sin v - V_N), \quad (23)$$

$$h_S = h_\infty + \frac{1}{2} (V_\infty^2 \sin^2 v - V_N^2), \quad (24)$$

where $v = \lambda + \delta - \alpha$. The constants $\alpha, V_\infty, \rho_\infty, p_\infty$, and h_∞ are input, and V_N is determined at each point by the Newton-Raphson method so that ρ_S, p_S , and h_S are consistent with the curve-fit equation of state of equilibrium air $\rho_S = \rho(p_S, h_S)$.

The function $\Delta = \Delta(Y) [0 < \Delta < \pi, \Delta(-Y) = \pi - \Delta(Y)]$ is defined as follows:

$$\text{for } |Y| \leq -R_1 \cos \Delta_1, \quad \Delta = \cos^{-1} \left(\frac{Y}{R_1} \right);$$

for $|Y| > -R_1 \cos \Delta_1$, $\Delta(Y)$ is either given in tabular

form as input ($Y > 0$), or, if a value of the input

$$\text{constant } R_2 \text{ is given, } \Delta = \cos^{-1} \left[\frac{Y + (R_1 - R_2) \cos \Delta_1}{R_2} \right] \text{ for } Y > 0.$$

Here R_1, R_2 , and $\Delta_1 (\pi > \Delta_1 > \Delta_0 > \pi/2)$ are input constants.

The following functions depend on $\Delta(Y)$:

$$R_n = \begin{cases} R_1 & \text{for } |Y| \leq -R_1 \cos \Delta_1, \\ R_2 & \text{for } |Y| > -R_1 \cos \Delta_1 \text{ and } R_2 \text{ given} \\ \frac{-1}{\frac{d\Delta}{dY} \sin \Delta} & \text{for } |Y| > -R_1 \cos \Delta_1 \text{ and } R_2 \text{ not given,} \end{cases} \quad (25)$$

$$R_\theta = \begin{cases} R_1 & \text{for } |Y| \leq -R_1 \cos \Delta_1, \\ \frac{Y}{\cos \Delta} & \text{for } |Y| > -R_1 \cos \Delta_1, \end{cases} \quad (26)$$

$$g = \begin{cases} R_1 \sin |\Delta_0 - \Delta| & \text{for } |Y| \leq -R_1 \cos \Delta_1, \\ \frac{Y \sin (\Delta_0 + \Delta_1)}{\cos \Delta_1} - Y R_1 \cos \Delta_0 \int_{-R_1 \cos \Delta_1}^Y \frac{dY}{Y^2 \sin \Delta} & \text{for } Y > -R_1 \cos \Delta_1, \\ \frac{Y \sin (\Delta_0 - \Delta_1)}{\cos \Delta_1} + Y R_1 \cos \Delta_0 \int_{R_1 \cos \Delta_1}^Y \frac{dY}{Y^2 \sin \Delta} & \text{for } Y < R_1 \cos \Delta_1, \end{cases} \quad (27)$$

Equations (13)-(15) are solved for $Y \geq R_1 \cos \Delta_0$, $U \geq 0$ and for $Y \leq R_1 \cos \Delta_0$, $U \leq 0$. At the initial point $\Delta = \Delta_0$, $U_{s_0} = 0$, which determines λ_0 from equation (20), and $S_b = S_{s_0} = S(p_{s_0}, h_{s_0})$ from equations (23) and (24) and the curve-fit equation of state. Also $U_{b_0} = 0$ at this point. The two unknowns are Δ_0 and ϵ_0 .

Since $g_0 = 0$, equation (13) has an indeterminate form at the initial point. The initial behavior of the function U_\pm is computed from the equation

$$\left[\rho \frac{dU}{dY} \right]_{b_0} = - \left[\rho \left(1 + \frac{\epsilon}{R_1} \right) \left\{ \frac{dU}{dY} \pm \frac{1}{2 \sin \Delta} \left| \frac{dY}{dY} \right| + \frac{v}{\epsilon \sin \Delta} \left(1 + \frac{\epsilon}{R_1} \right) \right\} \right]_{s_0} \\ (+ \text{ for } Y < R_1 \cos \Delta_0, - \text{ for } Y \geq R_1 \cos \Delta_0). \quad (28)$$

There will be two points, one in each direction, at which either $F = 0$ or $|u_k| \rightarrow a_k$ (whichever happens first). The location of these points is determined in the following manner:

- 1) If $F \rightarrow 0$, the integration is carried out to the point ($Y = Y_c$) where $F = 0$ and stopped. The quantity

$$d_1 = \left(1 - \frac{u_k^2}{a_k^2} \right)_{F=0} \quad (29)$$

is computed.

- 2) If $|u_k| \rightarrow a_k$, the integration is stopped at the point ($Y = Y_c$) where $1 - \frac{u_k^2}{a_k^2} = k$, a preset constant. The parameters \bar{Y} , k , and C are determined from the equation

$$1 - \frac{u_k^2}{a_k^2} = k \sqrt{|\bar{Y} - Y|} + C (\bar{Y} - Y) \quad (30)$$

evaluated at the last three points of integration.

The quantity

$$d_2 = (F)_{Y=\bar{Y}} \quad (31)$$

is computed by extrapolation of F to $Y = \bar{Y}$.

Initial estimates of Δ_o and ϵ_o are given by

$$\Delta_o^{(1)} = \frac{\pi}{2} + \alpha, \quad \epsilon_o^{(1)} = \frac{\frac{\rho_\infty}{\rho_{s0}} R_1}{1 + \sqrt{2 \frac{\rho_\infty}{\rho_{s0}}}} \quad (32)$$

The differential equations are integrated and the two parameters

$d_{i\pm}$ are evaluated ($i = 1, 2$ as described above; "+" refers to $Y > R_1 \cos \Delta_o$, "-" to $Y < R_1 \cos \Delta_o$). Next, a numerical minimization procedure is employed to find the roots of $d_{i\pm}$ as

functions of Δ_0 and ϵ_0 . Finally, the solution pertaining to the correct values of Δ_0 and ϵ_0 is determined.

REFERENCES

1. "Theoretical Prediction of the Equilibrium Real Gas Flow Fields about Blunt Bodies", Phase II Formal Written Report, Avco-Everett Research Laboratory, June 1963.
2. Hayes, W. and Probstein, R., Hypersonic Flow Theory. Academic Press, 1959.
3. Zlotnick, W. and Newman, D., "Theoretical Calculation of the Flow on Blunt-Nosed Axisymmetric Bodies in a Hypersonic Stream", Avco RAD TR-2-57-29, 1957.
4. Vaglio-Laurin, R. and Ferri, A., "Theoretical Investigation of the Flow Field About Blunt-Nosed Bodies in Supersonic Flight", J. Aerospace Sci., Vol. 25, 761, 1958.
5. VanDyke, W., "The Supersonic Blunt-Body Problem - Review and Extension", J. Aerospace Sci., Vol. 25, p. 485, 1958.
6. Mangler, K., "The Calculation of the Flow Field Between a Blunt Body and the Bow Wave", in Hypersonic Flow (Collan and Tinkler, eds.), Academic Press, 1960.
7. Garabedian, P., "Numerical Construction of Detached Shock Waves", J. Math. Phys., Vol. 36, p. 192, 1957.
8. Garabedian, P. and Lieberstein, H., "On the Numerical Calculation of Detached Bow Shock Waves in Hypersonic Flow", J. Aero. Sci., Vol. 25, p. 109, 1958.
9. Doby, R., "Equilibrium Air Hypersonic Axially Symmetric Flow Field Determination for Blunt Bodies", Avco RAD TM-63-49, July 1963.
10. Gravalos, F., Edelfelt, I., and Emmons, H., "The Supersonic Flow About a Blunt Body of Revolution for Gases at Chemical Equilibrium", 9th Annual Congress, Int'l Astronautical Fed., 1958.
11. Swigart, R., "A Theory of Asymmetric Hypersonic Blunt-Body Flows", AIAA Journal, Vol. 1, No. 5, p. 1034, 1963.
12. Belotserkovskii, O., "Flow Past A Circular Cylinder with a Detached Shock Wave", Vychislitel'naia Matematika, Vol. e, p. 149, 1958 (translated by S. Adashko, ed. by M. Holt, Avco RAD-9-TM-59-66, 1959).

13. Belotserkovskii, O., "The Calculation of Flow Over Axisymmetric Bodies with a Decaying Shock Wave", Academy of Sciences, USSR Computation Center Manograph, 1961 (translated by J.F. Springfield, Avco RAD-TM-62-64, 1962).
14. Dorodnitsyn, A., "On a Method for the Numerical Solution of Certain Problems of Aero-Hydrodynamics", Academy of Sciences, USSR, Proceedings of 3rd All-Soviet Math. Congress, Vol. 2 (1956), Vol. 3 (1958).
15. Traugott, S., "An Approximate Solution of the Direct Supersonic Blunt-Body Problem for Arbitrary Axisymmetric Shapes", J. Aerospace Sci., Vol. 27, No. 5, p. 361, 1960.
16. Holt, M., "Direct Calculation of Pressure Distribution on Blunt Hypersonic Nose Shapes with Sharp Corners", J. Aerospace Sci., Vol. 28, No. 11, p. 872, 1961.
17. Bazzhin, A., "The Calculation of Supersonic Flow Past A Flat Plate with a Detached Shock Wave", (translated by R.F. Probstein, Avco RAD TM to be published).
18. Minailos, A., "On the Calculation of Supersonic Flow Past Blunted Bodies of Revolution at Angles of Attack", (translated by R.F. Probstein, Avco RAD TM to be published).
19. Vaglio-Laurin, R., "On the PLK Method and the Supersonic Blunt-Body Problem", J. Aerospace Sci., Vol. 29, No. 2, p. 185, 1962.
20. Struick, D., Lectures on Classical Differential Geometry, Addison-Wesley, 1950.

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1. In the second paragraph on p. 11, change "r" to "n" in the fourth sentence.
2. Replace pp. 8-10 by the following:

III. CONFIGURATION OF THE DIVIDING STREAMLINE

Consider a cylindrical coordinate system (n, r, θ) with velocity components (v, u, w) , located at the stagnation point with n - axis perpendicular to the surface (Figure 1).

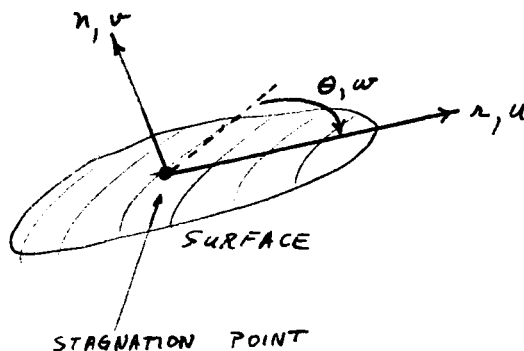


Figure 1. Configuration of Coordinates at the Stagnation Point

We assume that it is possible to expand the velocity components in Taylor series in r and n in the vicinity of the stagnation point:

$$\begin{aligned} v &= f(\theta)r + c(\theta)n, \\ u &= a(\theta)r + d(\theta)n, \\ w &= b(\theta)r + e(\theta)n. \end{aligned} \tag{1}$$

But $f(\theta) = 0$ because $v = 0$ at $n = 0$, and $c(\theta) = c = \text{constant}$ because otherwise v would not be single-valued along the n - axis.

The vorticity is given by

$$\vec{\Omega} = \left[\frac{(r\omega)_n - u_\theta}{r}, \frac{v_\theta}{r} - \omega_n, u_n - v_n \right] \quad (2)$$

in the (n, r, θ) directions, or

$$\vec{\Omega} = \left[2b - a_\theta + (c - d_\theta) \frac{n}{r}, -c, d \right]. \quad (3)$$

The momentum equation $(\vec{q} \cdot \nabla) \vec{q} = -\frac{1}{\rho} \nabla p$ gives $p_n = p_r = p_\theta = 0$ because of the vanishing velocity, and similarly the equation of conservation of stagnation enthalpy $h + \frac{1}{2} \vec{q}^2 = H$ (constant) gives $h_n = h_r = h_\theta = 0$. Hence all thermodynamic variables -- in particular, the density -- are stationary at the stagnation point by reason of the equilibrium equation of state. The continuity equation then gives

$$\rho \nabla \cdot \vec{q} + \vec{q} \cdot \nabla \rho = 0,$$

or, since $\nabla \rho = 0$,

$$\nabla \cdot \vec{q} = 2a + b_\theta + c + (d + e_\theta) \frac{n}{r} = 0,$$

thus

$$\begin{aligned} 2a + b_\theta + c &= 0, \\ d + e_\theta &= 0. \end{aligned} \quad (4)$$

The vorticity equation is written in the form $\nabla \times (\vec{q} \times \vec{\Omega}) = \nabla \times \left(\frac{1}{\rho} \nabla p \right)$.

Since the right-hand side vanishes,

$$\nabla \times (\vec{q} \times \vec{\Omega}) = (\xi, \eta, \zeta) = 0, \quad (5)$$

where

$$\begin{aligned}\xi &= - \left[(2b-a_\theta)(2a+b_\theta) - (2b-a_\theta)_\theta b \right] - \left[ce + (2b-a_\theta)d \right. \\ &\quad \left. + (e-d_\theta)a + \{ (e-d_\theta)b + (2b-a_\theta)e - cd \}_\theta \right] \frac{n}{\lambda} - \left[(e-d_\theta)e \right]_\theta \left(\frac{n}{\lambda} \right)^2 \\ &\equiv \xi_0 + \frac{n}{\lambda} \xi_1 + \left(\frac{n}{\lambda} \right)^2 \xi_2, \\ \eta &= (be)_\theta + (a+c)e + 2bd + 2e(d+e_\theta) \frac{n}{\lambda} \equiv \eta_0 + \frac{n}{\lambda} \eta_1, \\ \int &= (2b-a_\theta)e - bd_\theta - (a+c)d + 2e(e-d_\theta) \frac{n}{\lambda} = \int_0 + \frac{n}{\lambda} \int_1.\end{aligned}$$

Using equation (4), $\eta_0 = 0$ yields

$$bd - ae = 0 \quad (6)$$

The equation $\int_0 = 0$ becomes

$$2b(e-d_\theta) = 0,$$

so that

$$e - d_\theta = 0. \quad (7)$$

Now $(2b - a_\theta)$ must be constant in order that the vorticity component in the n -direction (equation (3)) be single-valued. The equation $\xi_0 = 0$ then implies

$$2b - a_\theta = 0 \quad (8)$$

Differentiating equation (8) with respect to θ and using the first of equations (4) to eliminate b_θ , we obtain the following differential equation for a :

$$a_{\theta\theta} + 4a + 2c = 0. \quad (9)$$

Let $a(0) = A$, $b(0) = 0$ (assuming a plane of symmetry), $a(\frac{\pi}{2}) = B$ (A and B are the velocity gradients in two orthogonal directions away from the stagnation point on the surface). Then equation (9) has the solution

$$a = \frac{A+B}{2} + \frac{A-B}{2} \cos 2\theta, \quad (10)$$

where $c = -(A + B)$ and equation (8) yields

$$b = -\frac{A-B}{2} \sin 2\theta \quad (11)$$

for the cross-flow gradient.

The two-dimensional stagnation point is represented by the special case $B = 0$. In general, the stagnation point has two axes of symmetry on the surface ($\theta = 0, \frac{\pi}{2}$).

Examination of equation (7) and the second of equations (4) in the same way yields

$$d = k_1 \sin \theta + k_2 \cos \theta,$$

$$e = -k_2 \sin \theta + k_1 \cos \theta.$$

At $\theta = 0$, equation (6) implies that $e(0) = 0 = k_1$. If in addition $B \neq 0$, then evaluation of equation (6) at $\theta = \frac{\pi}{2}$ gives

$$d = e \equiv 0$$

for all θ . If the stagnation point is two-dimensional, on the other hand, this conclusion cannot be drawn.

In a three-dimensional flow, therefore, there are two alternatives. First, the flow in the neighborhood of the stagnation point is analytic

(i.e., can be expanded in power series). Then unless $B = 0$, all three components of the vorticity at the stagnation point are zero, and the velocity components in the neighborhood of the stagnation point have the form

$$\begin{aligned}u &= a(\theta)r, \\w &= b(\theta)r, \\v &= cn,\end{aligned}\tag{12}$$

where a and b are given by equations (10) and (11). The outflow velocity u and the cross flow w vanish along the axis perpendicular to the body, and the stagnation streamline is perpendicular to the surface. If $B = 0$, however (two-dimensional flow), this is not so, and the stagnation streamline subtends an angle to the surface which is dependent on the way it passes through the shock rather than on conditions near the stagnation point.

The second possibility is that the flow does not have an analytic solution near the stagnation point. This possibility is rejected in the present context because it is inconsistent with the assumption that the flow variables can be approximated by analytic functions.